$$\Theta_*(\delta) = \Theta_0 - \Theta_1/(\delta + s_1),$$
$$\Theta_1 = \Theta_0^2 s_0^{-2}(k_0+1)^{-1}\left[\frac{\Theta_0 k_0}{k_0+1} - 2\right]\left[\frac{1}{\Theta_0} - \frac{\Theta_0 k_0 \xi}{(k_0+1)^2}\right]^{-1}, \quad (4.2)$$
$$s_1 = s_0\Theta_0\left(\frac{1}{\Theta_0} - 1 - \frac{\xi}{k_0+1}\right) - \frac{\Theta_0/s_0^2}{k_0+1}.$$

The value of Θ_0 is found by solving the transcendental equation

$$\Theta_0 = [1 + \xi/(k_0+1)]^{-1}.$$

In practice, (4.1) and (4.2) describe $\sigma_*(\delta)$ and $\Theta_*(\delta)$ satisfactorily for $\delta \geqslant 0.2$ (error less than 0.5%).

Figures 1-3 have been given for the case $\beta = 0$ and $\xi = 1$, but incorporating $\beta \neq 0$ does not result in any substantial quantitative changes in the curves (the maximum relative deviation does not exceed β).

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MOTION OF A CLOUD OF HEATED PARTICLES ABOVE A HORIZONTAL SURFACE IN AN EXTERNAL FORCE FIELD

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UDC 532.529 + 536.46

The motion of a system (cloud) of particles in an external force (gravity) has been studied experimentally and theoretically in the isothermal case where the temperatures of the particles and carrier medium are the same; a review and bibliography is given in [1, 2]; see also [3, 4]. One of the basic features of these studies was the identification of two different types of motion of the cloud depending on the degree of hydrodynamical or gas-dynamical interaction between particles via the carrier phase. In the "filtration" regime this interaction is small and each particle in the cloud moves independently. In the "entrainment" regime, because of the friction between the phases, large-scale motion (of the order of the size of the cloud) of the dispersed medium with a rising flow on the periphery arises and the precipitating cloud is transformed into a vortex ring with continuously increasing diameter.

Many phenomena in nature and in technological processes are accompanied by the formation of aerosol clouds in which the temperature of the particles is higher than that of the ambient medium (emission from a smokestack, combustion products in fires, emission of aerosols in the eruption of volcanoes). The initial temperature differential leads to new features in the evolution of the cloud of particles. As a result of interphase heat exchange the gas in the cloud is heated and expands, carrying along particles with it. As a result, for a sufficiently high concentration of particles, the cloud size increases in the initial

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stage of the process (cloud dispersion) and the volume content of particles correspondingly decreases. Another important effect is that along with the precipitation of particles under the action of the external force, there arises motion of particles in the opposite direction as a result of thermal convection of the heated gas. For a sufficiently large initial temperature differential, the carrying away of particles by the rising current of gas can be so significant that the cloud ruptures and divides into two smaller clouds moving in opposite directions.

In the present paper, using the equations of mechanics for a two-phase medium [5] describing the system in terms of two interacting and interpenetrating continua (gas and particles), and the methods of numerical integration, we describe the motion of a cloud of heated particles of identical size in an external force field above a horizontal surface (surface of precipitation). The evolution of the cloud of particles is followed, estimates for several quantities characterizing the dispersion of the cloud are given, and the condition for cloud division is determined. The possibility of using the results to describe the aerodynamics of a cloud of slowly burning particles is noted.

1. We consider an initially cold gas at temperature T_0 in static equilibrium in an external force field above a plane horizontal surface in which there is a cloud of solid or liquid spherical particles of identical size (a monodisperse aerosol). The spatial extent of the cloud in one of the horizontal directions is much larger than in the other. We will consider the problem in the planar formulation, introducing the coordinate system (x, y) in a plane perpendicular to the long axis of the cloud with an origin lying in the plane of precipitation under the center of gravity of the cloud.

The initial conditions are

$$t = 0,\ T_1 = T_0,\ \mathbf{U}_i = 0,\ \rho_1 = \rho_{10}\exp(-gy/R_0T_0),\quad p = \rho_1 R_0 T_0,$$
$$T_2 = T_0 + T_s,\quad n = n_0 \exp[-(x^2 + (y-H)^2)/R^2],\quad \rho_2 = \rho_2^0 n\pi d^3/6, \tag{1.1}$$

where subscripts 1 and 2 refer to gas and particles, respectively, and t is the time, ρ_i, $\mathbf{U}_i(u_i,\ v_i)$, $T_i\,(i = 1, 2)$ are respectively the mean density, velocity, and temperature of the phases, n is the number of particles per unit volume, $T_0 + T_s$ is the initial temperature of the particles, g is the acceleration of the external force directed along the normal to the plane in the direction of negative y, R_0 is the gas constant, ρ_2^0 is the intrinsic density of the particles, d is the particle diameter, H is the initial height of the cloud, and p is the pressure of the gas.

We introduce dimensionless variables, choosing as scales of measurement the initial radius of the cloud R, the velocity $(gR)^{1/2}$, the time $(R/g)^{1/2}$, the temperature T_0, the concentration of particles n_0 in the center of the cloud at t = 0, the pressure $\rho_{10}R_0T_0$, the density of gas ρ_{10} near the precipitation surface; in addition, the dimensionless quantities entering (1.1) are used below in the same notation. We assume that the volume content of particles is small $\alpha_2 \ll \alpha_1$, and the ratio of intrinsic densities of gas and particles $\varepsilon = \varepsilon = \rho_{10}/\rho_2^0 \ll 1$. We ignore fragmentation and evaporation of particles and assume that the temperature over the volume of a particle is constant. Then the plane nonstationary motion of the cloud of particles in the gas is described by the following equations in dimensionless variables:

$$\dot{\rho}_1 = -\rho_1 \operatorname{div}\mathbf{U}_1,\ p = \rho_1 T_1,\ \rho_1\dot{T}_1 = \gamma(\mathrm{RePr})^{-1}\Delta T - (\gamma-1)p\operatorname{div}\mathbf{U}_1 + q, \tag{1.2}$$
$$\rho_1\dot{\mathbf{U}}_1 = -(\gamma \mathrm{M}^2)^{-1}\operatorname{grad} p + \rho_1\mathbf{g} - \mathbf{f} + \mathrm{Re}^{-1}(\Delta\mathbf{U}_1 + (1/3)\operatorname{grad}\operatorname{div}\mathbf{U}_1);$$
$$\dot{\rho}_2 = -\rho_2\operatorname{div}\mathbf{U}_2,\qquad \rho_2\dot{T}_2 = -\gamma_1 q,\quad \rho_2\dot{\mathbf{U}}_2 = \rho_2\mathbf{g} + \mathbf{f},\quad \rho_i = \rho_i^0\alpha_i \tag{1.3}$$
$$(\dot{\varphi}_i = [\partial/\partial t + (\mathbf{U}_i\operatorname{grad})]\varphi_i,\ \Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2,\ \mathbf{g} = (0, -1));$$
$$\mathrm{M}^2 = Rg/\gamma R_0T_0,\ \mathrm{Re} = R^{3/2}g^{1/2}\rho_{10}/\eta,\ \mathrm{Pr} = c_p\eta/\lambda,\ \gamma = c_p/c_V,\ \gamma_1 = c_p/c_2, \tag{1.4}$$

where M, Re, and Pr are the Mach, Reynolds, and Prandtl numbers, η and λ are the dynamical viscosity and thermal conductivity, respectively, with both assumed to be constants, c_p and c_V are the heat capacities of the gas at constant pressure and volume, and c_2 is the heat capacity of the particles. The external force and the heat exchange between the phases is taken into account in (1.2) and (1.3) by introducing the following exchange terms

$$\mathbf{f} = 3\varepsilon c_d \rho_1 \rho_2 |\mathbf{W}| \mathbf{W}/4\delta r, \quad c_d = 24(1 + 0.158\mathrm{Re}_p^{2/3})/\mathrm{Re}_p, \quad \delta = d/R, \tag{1.5}$$

$$\mathrm{Re}_p = \mathrm{Re}_p^0 \rho_1 |\mathbf{W}| r^{-1}, \quad \mathrm{Re}_p^0 = d(Rg)^{1/2}\rho_{10}/\eta \, (\mathbf{W} = \mathbf{U}_1 - \mathbf{U}_2, \quad r = 1 - \alpha_2^0 n);$$
$$q = 6\gamma\varepsilon \, \mathrm{Nu} \, \rho_2 (T_2 - T_1)/\delta \, \mathrm{Pr} \, \mathrm{Re}_p^0, \quad \mathrm{Nu} = 2 + 0.6\mathrm{Pr}^{1/3}\mathrm{Re}_p^{1/2}, \tag{1.6}$$

where c_d is the drag coefficient, Re_p is the instantaneous value of the Reynolds number of the particles, Re_p^0 is the Reynolds number of the particles computed using the characteristic convective velocity, Nu is the Nusselt number characterizing the magnitude of the heat exchange between particles and the gas, $\alpha_2^0 = n_0 \pi d^3/6$ is the maximum volume fraction of particles at the initial instant of time.

The initial conditions (1.1) and the boundary conditions taking into account the symmetry of the problem with respect to the plane x = 0, the static equilibrium of the gas at infinity, the "cohesion condition" for the gas velocity at the precipitation surface (which is assumed to be adiabatic) have the form

$$t = 0, \; T_1 = 1, \; \mathbf{U}_i = 0, \; \rho_1 = \exp(-\gamma \mathrm{M}^2 y), \; p = \rho_1, \; T_2 = 1 + \theta$$
$$(\theta = T_s/T_0 > 0), \quad n = \exp[-x^2 - (y - H)^2], \quad \rho_2 = \alpha_2^0 n/\varepsilon;$$
$$x^2 + y^2 \to \infty, \; \mathbf{U}_1 = 0, \; T_1 = 1, \; \partial p/\partial y = -\gamma \mathrm{M}^2 \rho_1; \; x = 0, \; u_i = 0, \tag{1.7}$$
$$\partial v_i/\partial x = \partial \rho_i/\partial x = \partial T_i/\partial x = 0; \; y = 0, \; \mathbf{U}_1 = 0, \; \partial T_1/\partial y = 0.$$

It is assumed that the particles reaching the precipitation surface remain there.

We consider a qualitative estimate. For an aerosol cloud of radius ~1 m in which particles of diameter ~10^{-4} m are suspended, for g = 9.8 m/sec^2, $\eta/\rho_{10} = 10^{-5}$ m^3/sec, we have $\mathrm{Re}_p^0 \sim 10$, Re ~ 10^5. The large value of the "external" Reynolds number indicates that the motion of the cloud will be turbulent; this is accounted for in the present paper by giving the "external" Reynolds and Prandtl numbers evaluated with the effective turbulent transport coefficients. Also one assumes that the particle diameter is small in comparison to the characteristic spatial scale of the turbulence, the streamlining of particles is viscous, and for the internal Reynolds and Prandtl numbers in (1.6), (1.7) we use the molecular transport coefficients. Then Re and Re_p^0 will be independent parameters.

In the calculations the following parameters were held constant: $\mathrm{M}^2 = 0.75 \cdot 10^{-3}$, Re = 29,05, Pr = 1, $\varepsilon = 10^{-3}$, $\gamma = 1{,}4$, $\gamma_1 = 1$ and the other parameters were varied as follows: $\delta = 3.3 \cdot 10^{-5} - 1.4 \cdot 10^{-4}$, $\alpha_2^0 = 10^{-5} - 10^{-2}$, $\theta = 0 - 5$, $\mathrm{Re}_p^0 = 6.5 \cdot 10^6 \delta$, $H = 1.5 - 10$.

The solution of (1.2) through (1.7) is obtained by numerical integration applying the method of [6, 7] to the equation of motion of the gas and the longitudinal-transverse trial run method of [8] to the equation of motion of the particles. The calculations were done on a nonuniform grid with respect to both axes and with time steps given by $\tau = \mathrm{KM}h_m$, where K = 4 is the Courant number and h_m is the minimum spatial step of the grid. A detailed discussion of the methods used in the calculation can be found in [2].

2. In the isothermal case (θ = 0) the motion of the carrier medium is due to the precipitation of particles; under the action of the external force the particles begin to move downward and they carry along the surrounding gas. This motion of the carrier medium occurs for sufficiently large particle concentrations where the friction between phases is large. The solution of the planar problem in this case leads to two cylindrical vortices increasing in size and symmetric with respect to the plane x = 0 [2]. In the plane of symmetry, the gas in the cloud moves downward while on the periphery it moves upward. For a small concentration of particles the carrier medium is practically at rest and the particles move as independent units, "filtering" through the fixed medium.

The nonisothermal case (θ > 0) differs in that at the intial instant of time there is a temperature differential between the particles and gas. As a result of the equalization of temperatures of the two phases, the gas is heated. Therefore in the nonisothermal case, besides the motion of the carrier medium caused by precipitation of particles, another type of motion can be observed connected with the expansion of the heated gas and the development of natural thermal convection. Because particles are drawn into these motions, the cloud undergoes a significant change in comparison with the isothermal case. This refers to the entrainment regime, considered below. For a sufficiently small particle concentration (filtration regime) the heating of the gas is significant and the cloud precipitates as in the isothermal case [2]. For illustration of the features of the cloud motion in the nonisothermal case, refer to Fig. 1 where the time dependence of the maximum (over space) velocities $U_{im} = \max|U_i|$ (solid curves) and temperature $T_{im} = \max T_i$ (dash-dot curves) are

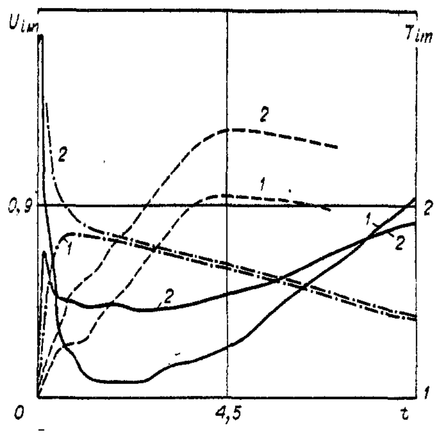

Fig. 1

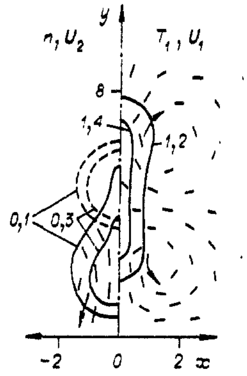

Fig. 2

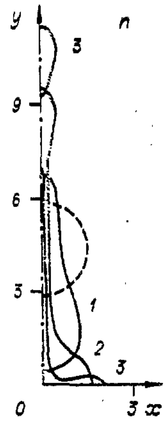

Fig. 3

shown for the two phases where $\theta = 2$, $\alpha_2^0 = 10^{-3}$, $\delta = 6.67 \cdot 10^{-5}$, $H = 5$. The enumeration of the curves corresponds to that for the phases and the dashed lines give $U_{im}(t)$ for $\theta = 0$. The maximum values of the velocities and temperatures are reached in the plane of symmetry.

In the isothermal case when the cloud is falling, the velocity of the particles U_{2m} is larger than the velocity of the gas U_{1m}. The difference in the velocities results from the fact that in the plane of symmetry, where the velocities are maximum and $u_i = 0$, the velocity of particles relative to the carrier medium, over a time of order of the velocity relaxation time τ_r, is approximately equal to the velocity of motion w of the separate particles. For the parameters used in Fig. 1, integration of the equation of motion of the independent particles [2] gives $\tau_r = 0.54$, $w = 0.31$. Large-scale vortex motion is formed after a time corresponding to the displacement of the cloud by a distance of the order of its initial size $t \sim 1/w \simeq 3.2$.

In the nonisothermal case in the early stages of evolution of the cloud, an increase in the maximum velocity of the gas ($v_{1m} \simeq 1.64$ at $t \simeq 0.09$) is observed; this is due to expansion of the heated gas. The expanding gas moves from the center of the cloud in all directions, carrying particles along with it. This motion of the particles is superimposed on the motion downward under the external force. A maximum in the velocity $v_{2m}(t \simeq 0.2)$ occurs (this is absent in the isothermal case) resulting from the dispersion of the cloud in response to the expanding gas.

We now estimate some characteristics of the dispersion of the cloud.

The dispersion time of the cloud, measured from the initial instant of time, is determined by the characteristic time of the slowest stage of the process, which is conductive heating of the gas. Therefore it can be estimated as the temperature equalization time between the phases $t_1 \simeq l^2/4$ where $l = \langle n \rangle^{-1/3}$ is the mean distance between particles, $\langle n \rangle = kn_0$ is the mean concentration of particles over the cloud determined by integrating the initial distribution n(x, y) over the volume of the cloud (for a cloud with a uniform distribution of particles k = 1), and finally $\varkappa = \lambda/\rho_{10}c_p$ is the thermal conductivity of the gas.

Writing n_0 in terms of the volume fraction of the particles α_2^0 and the particle diameter we have

$$\tau_1 = t_1 (g/R)^{1/2} \approx (1/4)\left(\pi/6\alpha_2^0\right)^{2/3} \mathrm{Re}_p^0 \delta \mathrm{Pr}. \tag{2.1}$$

One can estimate the average temperature of the gas and particles after dispersion of the cloud using the balance of thermal energy given off by the particles and acquired by the gas:

$$c_2 M_2 (T_s - \langle T \rangle) = c_p M_1 (\langle T \rangle - T_0), \quad M_1 \approx \rho_{10} \pi R^2, \quad M_2 = k \alpha_2^0 \rho_2^0 \pi R^2,$$

where M_i is the total mass of the i-th phase in the cloud. We have

$$\langle \theta \rangle = \langle T \rangle / T_0 = 1 + [\theta/(1+\sigma)], \quad \sigma = c_p M_1 / c_2 M_2 = \gamma_1 \varepsilon / k \alpha_2^0, \tag{2.2}$$

so that the average temperature is determined by the ratio of the total mass of gas and particles contained in the cloud. Therefore the particles can heat the gas, despite their small volume content.

The expansion of the gas can be considered as an isobaric process in which the parameters of the gas satisfy the relation $V_0/T_0 = V_1/T_1$ (V_0 and V_1 are the volumes of the cloud at $t = 0$ and $t = t_1$). It then follows that the volume of the cloud increases by a factor of $\langle\theta\rangle$ and the radius R_1/R by $\langle\theta\rangle^{1/2}$ as a result of dispersion. The mean velocity of the gas is given by $\langle u\rangle = (\langle\theta\rangle^{1/2} - 1)/\tau_1$.

For the parameters used in Fig. 1, the above estimates are $\tau_1 = 0.64$, $\langle\theta\rangle = 1.77$, $R_1/R = 1.33$, $\langle u\rangle = 0.51$, which compares well with the numerical results $\tau_1 = 0.6$, $\langle\theta\rangle = 2$, $R_1/R = 1.21$ (the radius of the cloud is defined with respect to the line of constant concentration $n = 0.1$). Comparing the values of τ_1, $\langle\theta\rangle$, and R_1/R obtained using (2.1), (2.2) with those of the numerical calculations for different values of the parameters shows that the above estimates give the correct dependence on the parameters α_2^0, δ and θ.

The greater the degree of temperature equalization between the two phases (the temperature difference is 10% of the initial temperature differential for $T_1 \simeq 0.6$; see the dash-dot curve in Fig. 1), the smaller the velocity of the gas, and the particles are displaced only downward.

From this point up to time $t \simeq 2$ the cloud precipitates in the external force field. The velocities of gas and particles will be much smaller than in the isothermal case due to the decrease in the volume fraction of particles in the initial stage of the process from the increase in the cloud size because of dispersion. This leads to a decrease in the gas-dynamic interaction between particles. The relative velocity of motion of the phases is $v_{2m} - v_{1m} \approx w$.

From time $t \approx 2$ on, thermal process significantly affect the motion of the cloud. The charaeteristic time for the development of thermal convection can be estimated from the formula $\tau_c = t_c(g/R)^{1/2} \quad [2R_1/R(\langle\theta\rangle - 1)]^{1/2} = 1.55$ which gives for the instant of time corresponding to the onset of convection the value $t = \tau_1 + \tau_c = 2.15$ which agrees with the numerical result. The gas velocity U_{1m} again begins to increase because of the development of thermal convection in the gas. The rise of heated gas is accompanied by the formation of vortex convective motion in the upper part of the cloud. This motion is in the form of a pair of symmetric cylindrical vortices. They can be called thermal vortices since they occur only in the non-isothermal case. This type of motion is observed in rising volumes of hot gas under the action of the buoyancy force [9]. An increase in the intensity of the buoyant vortices leads to an increase in the gas velocity, and it begins to exceed the velocity of the particles U_{2m} (this corresponds to the interaction of the solid curves in Fig. 1).

Almost simultaneously with the formation of thermal vortices, a large-scale vortex motion of both phases develops in the lower part of the cloud. This is caused by the precipitation of particles under the external force. This motion has been studied in the isothermal case in [2, 3]; it is characterized by a continuous increase in the downward velocity of the cloud of particles. Therefore in Fig. 1 the value U_{2m} increases until the majority of the particles reach the surface.

The development of vortex motion formed by upper thermal vortices and lower "sedimentation" vortices is shown in Fig. 2 for $t = 7.4$. The pattern is completely symmetric about the line $x = 0$. To the right of the axis the velocity field and isotherms of the gas are given; to the left are given the velocity field of the particles and the lines of equal concentration. Values are shown for several of the lines of equal T_1 and n, and the initial concentration distribution is shown by the dashed lines.

Because of the rise of the heated gas, the isotherms are drawn toward the vertical and eventually ($t \approx 11$) "rupture" with the upper part of the isotherm taking on the form of a buoyant mushroomlike thermal. The intensity of the thermal convection is not sufficient in this case to carry upward a sufficient number of particles. However the rising flow of gas on the symmetry axis retards the motion of particles in the upper part of the cloud. Therefore the lines of equal concentration are also drawn toward the vertical. The calculation stops when the thermal vortices leave the region of interest and the particles are completely precipitated on the horizontal surface.

3. For a sufficiently strong convective motion of the heated gas, a significant number of particles are trapped by the rising thermal vortices and are carried upward. At the same time the remaining particles continue moving downward under the external force. Hence the lines of equal concentration become strongly distended toward the vertical and eventually rupture with the cloud dividing in two. This process is shown in Fig. 3 for $\theta = 3$,

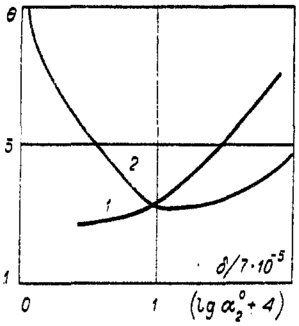

Fig. 4

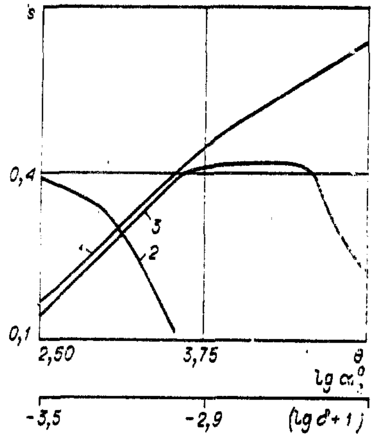

Fig. 5

$\alpha_2^0 = 10^{-3}$, $\delta = 6.67 \cdot 10^{-5}$, $H = 4.4$. The equal concentration line n = 0.1 is shown at t = 0 (dashed curve) and t = 7.4, 11.1, 13.8 (curves 1 through 3, respectively). According to the degree of cooling of the cloud its rise is slowed down. Eventually it must come to rest and precipitation of particles begins.

The condition for cloud division within the range of parameters chosen above basically depends on $\theta, \delta, \alpha_2^0$. In the numerical calculation we determine the boundary separating the different types of cloud motion for a fixed value of the cloud height H = 3.2. In Fig. 4 $\theta(\delta)$ is shown for $\alpha_2^0 = 10^{-3}$ (curve 1) and $\theta(\alpha_2^0)$ for $\delta = 10^{-5}$ (curve 2). Above these curves there is cloud division; below them the cloud moves as a whole. In these calculations it was assumed that in cloud division the thermal vortices carry away not less than 10% of the total number of particles.

An increase in the size of the particles for a fixed value of the total mass of dispersed material leads to a shift of the boundary toward higher initial temperature differentials (curve 1); this is a consequence of the greater inertia of large particles and the fact that a more intense thermal vortex is required to capture them. Curve 2 shows how the boundary dividing the solutions changes when we vary the particle concentration and keep other parameters fixed. An increase in particle concentration leads on the one hand to a greater heating of the gas in the cloud. This can increase the intensity of the thermal vortex and capture a greater number of particles in the rising current of gas. On the other hand, the absolute number of particles also increases (directly proportional to α_2^0) which must go into the rising cloud in order that there be a division. These two factors determine the nonmonotonic behavior of curve 2: For small α_2^0 the increase in the intensity of the thermal vortices is the most important factor; for $\alpha_2^0 > 10^{-3}$ the second factor dominates. When the initial height of the cloud is increased, the boundary dividing the solutions is displaced in the direction of larger θ because for small cloud heights the precipitation surface hinders its fall and therefore encourages cloud division.

In order to study the cloud division process quantitatively, we introduce a cloud division parameter given by the fraction of particles in the rising cloud: $s = N_b/N_0$ where N_b is the number of particles in the upper cloud and N_0 is the total number of particles. The calculations were carried out for conditions of cloud division and yielded s as a function of the initial temperature differential, and the size of the particles and their volume fraction. In Fig. 5 (H = 3.2) the dependence of s on θ is shown (curve 1: $\alpha_2^0 = 10^{-3}$, $\delta = 6.67 \cdot 10^{-5}$) as well as the dependence on δ (curve 2: $\alpha_2^0 = 10^{-3}$, $\theta = 3$) and α_2^0 (curve 3: $\delta = 6.67 \cdot 10^{-5}$, $\theta = 3.5$). With an increase in θ the intensity of thermal convection increases and therefore the function s(θ) increases with θ. The downward direction of curve 2 is explained by the fact that larger particles are more difficult to trap by the thermal vortices. The nonmonotonic behavior of curve 3 is due to the same causes as the curve $\theta(\alpha_2^0)$ in Fig. 4.

Finally we note that our results can be useful in aerodynamical calculations of clouds of burning particles suspended in a cold gas above a plane horizontal surface. The dispersion and division of the cloud, the characteristic gasdynamical pattern of ascending and descending flows can be realized if the combustion time of the particles is larger than the characteristic time for the development of convection in the gas and the precipitation time.

Calculations were done for large H and showed that the features of cloud evolution discussed above (Figs. 1-3) also apply when the cloud falls in an unbounded space; the curves shown in Figs. 4, 5 apply quantitatively to this case.

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TWO-PHASE THREE-COMPONENT FILTRATION WHEN OIL IS DISPLACED BY A SOLUTION OF AN ACTIVE ADDITIVE

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Among the new methods for increasing the output of oil from rock strata, an important place is occupied by processes in which the oil is displaced by solutions of active additives: carbon dioxide gas or surface-active substances. Self-similar solutions were obtained earlier for the case of frontal displacement of the oil by dilute solutions of the additives [1, 2]. At high concentrations of a pumped-in solution the transition of the additive from the injection phase to the oil phase leads to an increase in the mobility of the oil and has a substantial effect on the displacement process. In [3] solutions were obtained for the problem of forcing oil out with solutions of any concentrations, on the assumption that the total volume of the phase remained constant when dissolution took place, and we obtained a number of solutions for problems of frontal displacement. In the present study this system of equations is considered in connection with an active additive which can be dissolved in water and oil but does not cause interphase mass exchange between the water and oil components. We investigate the problem of the decomposition of an arbitrary discontinuity, and we obtain self-similar solutions for problems of frontal displacement with arbitrary values of flooding of the stratum and any forms of the distribution function of the additive between the phases. From the solution of the problem of the structure of the discontinuity, we obtain the conditions for stability of the generalized solution. We investigate typical interactions of simple waves and shock waves, and we obtain solutions for problems involving displacement of the oil by a dose of the solution of active additive forced through the stratum by water.

1. Analysis of the Initial System of Equations. In the displacement process the additive is distributed between the water and oil phases. The system of equations of a two-phase three-component filtration consists of the equation of discontinuity for the water component, the oil component, and the active component [5]. When we consider large-scale displacement processes, we disregard the capillary jump in pressure between the phases,

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